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A theoretical analysis of bubble interaction in boiling systems

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Abstract

Classical analyses of boiling nucleation typically employed a linear approach with nucleation phenomena investigated by focusing on a single site and the heat transfer obtained from the active site density assuming uniform wall superheat. Possible strong interactions among bubbles were ignored. This paper proposed a new theoretical framework to analyze the interactions among bubbles and the self-organizing effect among bubbles was revealed. These studies may provide a more reasonable mechanistic description or engineering instruction on boiling heat transfer.

Keywords: Boiling; Bubble; Nucleation; Nonlinear; Interaction

1. Introduction

Boiling is often encountered in a wide variety of applications, including traditional industrial processes, such as metallurgical quenching, flooded tube and shell evaporators, and immersion cooling of industrial components and modern heat transfer technologies used in space, electronic components, nuclear reactors, etc. The importance of boiling in a wide variety of applications has provided an incentive for numerous investigations of its mechanisms over the past several decades. A substantial number of such efforts have been devoted to understanding and modeling the heat transfer during the boiling process. A plethora of empirical correlations are now available in the literature [1]. However, because of the multiplicity of variables influencing the boiling systems and strong nonlinear features [2,3], a complete theory is still far from being created, because the phenomena are too complicated and have not been sufficiently understood.

In classical theories, boiling heat transfer predictions still remain principally an empirical art and traditional modeling efforts typically use a linearized approach. For example, the physical phenomena are analyzed on the basis of a single site or vapor bubble, and the heat transfer rate is obtained for a given active site density distribution by assuming uniform conditions, i.e., the bubbles have no effect on the formation of adjacent bubbles. Consequently, possibly important interactions between bubbles are ignored. However, for practical boiling processes, interactions do occur between adjacent bubbles [4]. Therefore, the traditional linear approach often conflicts with observations of hysteresis, intermittent activity, deactivation of sites with increasing heat flux and interactions among sites [5]. More severely, classical theory cannot be effectively related to macroscopic models because it is based on individual discrete active sites [6].

The bubbles or sites on the boiling surface are randomly distributed [7–9]. Nucleate bubbles mainly interact by two kinds of thermal and hydrodynamic processes [10–12] that change the local superheat and the other parameters that determine the stability of adjacent nuclei. As the local transient wall superheat values vary stochastically, nucleation sites will stochastically affect the on/off states of adjacent sites.

The development of modern measurement techniques has provided much visualization of boiling processes and then synergetic effects [11–13]. Kenning and Yan [11] noted that active sites can co-exist within one bubble radius of each other and that the sites interact through the variation in wall temperature induced by embryo bubble generation, which results in intermittent bubbles generation. Judd and Chopra [12] investigated different mechanisms for the interactions, which may also be interpreted as self-organizing phenomena described here. Nelson and Bejan [14] demonstrated that

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Nomenclature

$b_1 \sim b_4$	constants
f_i	functions introduced in Eq. (22)
F_u, F_s	fluctuation forces introduced in Eqs. (20) and (21)
G	function defined in Eqs. (15) and (38)
h	heat transfer coefficient, function defined in Eqs. (15) and (26) ..... $W{\cdot}m^{-2}{\cdot}K^{-1}$
J, $\overline{J}$	flux ..... W
J''	flux density ..... $W{\cdot}m^{-2}$
l	characteristic length ..... m
$\underline{L}$	matrix defined in Eq. (30)
$\vec{N}$	function defined in Eq. (36)
N_s	the number of stable modes
p	exponential coefficient
r	bubble radius ..... m
S	function defined in Eq. (20)
t	time ..... s
T	function introduced in Eq. (21)
u, v	vectors defined in Eqs.(28) and (35), respectively, velocity ..... $m{\cdot}s^{-1}$
$\mathbf{W}_s$, $\mathbf{W}_u$	potential functions of stable modes and unstable modes
x	driving force
$\mathbf{x}$	driving force vector
z	coordinates

Greek symbols

ρ	probability density
$\boldsymbol{\sigma}$, $\bar{\boldsymbol{\sigma}}$	constants
η	constants
λ	damping coefficient
α	controlling parameter defined in Eq. (43)
$\boldsymbol{\Phi}$, $\overline{\boldsymbol{\Phi}}$	potential functions
ζ	constant
ξ	normalized temperature difference variable
μ	constant
ψ	constant
Ω	area unit

Subscripts

0	reference state
S	saturation
s, s', s'', s'''	stable modes
u, u', u'', u'''	unstable modes

competing mechanisms exist among sites, and that only preferential sites will be activated, while other sites will be suppressed [15].

However, the vivid dynamics characteristics are not very clear yet. A new theoretical perspective on the boiling process is needed. Different from partial differential equations, here, new equations for boiling are obtained from statistical mechanics and a corresponding mathematical description of bubble interaction is elaborated. Self-organization phenomena are correspondingly investigated. Industrial instructions based on theoretical results are finally discussed.

2. Physical descriptions on bubble interactions—beyond nonlinear partial differential equations

2.1. Classical difficulties from nonlinear partial differential equations

The nonlinear effects involved in boiling systems include non-uniform site distribution, the on/off behavior of sites, the formation and evaporation of micro-layers and macro-layers, bubble generation, growth, departure and coalescence, and interactions between bubbles, etc. Of course, the nonlinear behavior of pool boiling systems would be investigated if we could solve the controlling partial differential equations in a control volume of the two-phase system in the boundary layer adjacent to the heater surface. However, problems rest with the fact that we have difficulties in dealing with this kind of nonlinear partial differential equations (though numerical computation can make some achievements): For one reason, we have not yet a general way to analytically deal with nonlinear partial differential equations in mathematical kingdom; For another reason, active sites or bubbles are stochastically distributed on a surface area. The very special boundary conditions make it difficult to obtain exact theoretical solutions for nonlinear partial differential equations. In a word, an alternative theoretical framework is highly needed to describe the bubble interaction process.

2.2. Statistical mechanics analysis of bubble interaction—beyond nonlinear partial differential equations

The equivalence of Lagrange's analytical mechanics to Newton's framework for mechanics (partial differential equation for mechanics) gives us hint that we can evade partial differential equation in an alternative way when dealing with boiling systems. Statistical mechanics based on Lagrange's optimization may be an alternative way. The reasons are as follows:

Firstly, descriptions of boiling need statistical mechanics. In the boiling process, the parameters of heater surface, such as temperature are extremely non-uniform and rapidly vary with time for the nonlinear interactions [8]. It is necessary to analyze nonlinear characteristics from a microscopic view. Macroscopic complex boundary systems adjacent to the heater surface consist of a large number of objects, viewed as elementary. Doubtless, a sufficiently complete and adequate

dynamic description in most cases can only be achieved on the basis of the statistical mechanics.

In addition, we need Lagrange's optimization. For problems in thermal equilibrium of closed systems, entropy is maximized as an extreme. By dealing with entropy, we can obtain a microscopic understanding of thermal equilibrium closed systems. For non-equilibrium open systems, flux may be chosen a maximized property [16]. Maximization of flux can be developed by Lagrange's optimization [4,17]. Then let us discuss how Lagrange's optimization can bring new understanding of microscopic nonlinear dynamics of boiling.

Assuming the driving forces of sub-elements, i.e., the cause of receiving flux from heater, can be expressed as $x_1, x_2, \ldots, x_n$, which are lumped as a vector $\mathbf{x} = (x_1, x_2, \ldots, x_n)$, the driving forces here indicate temperature difference between heater and liquid bulk. Similar to classical statistical theory, here we can consider that all possible microstates compose a continuous range in the Γ space. $\mathrm{d}\mathbf{x} = \mathrm{d}x_1\mathrm{d}x_2\cdots\mathrm{d}x_n$ is a volume unit in Γ space. The probability for the state of the system existing within the volume unit $\mathrm{d}\mathbf{x}$ at time t is

$$\rho(\mathbf{x}, t)\,\mathrm{d}\mathbf{x} \tag{1}$$

$\rho(\mathbf{x}, t)$ is distribution function of ensemble, which satisfies the normalization condition

$$\int \rho(\mathbf{x}, t)\,\mathrm{d}\mathbf{x} = 1 \tag{2}$$

Assuming that the heat flux is J when the state of the system exists within the volume unit $\mathrm{d}\mathbf{x}$ at time t, the averaged flux over all possible microstates is

$$\overline{J} = \int \rho(\mathbf{x}, t) J(\rho)\,\mathrm{d}\mathbf{x} \tag{3}$$

By use of Lagrange multiplier, let us maximize Eq. (3) under the following constraints (i.e., prices given)

$$\langle x_i \rangle = b_1 \tag{4}$$

$$\langle x_i x_j \rangle = b_2 \tag{5}$$

$$\langle x_i x_j x_k \rangle = b_3 \tag{6}$$

$$\langle x_i x_j x_k x_l \rangle = b_4 \tag{7}$$

We obtain that [4,17]

$$\rho = \exp\Bigg\{\zeta + \sum_i \sigma_i x_i + \sum_{ij} \sigma_{ij} x_i x_j + \sum_{ijk} \sigma_{ijk} x_i x_j x_k + \sum_{ijkl} \sigma_{ijkl} x_i x_j x_k x_l + \cdots\Bigg\} \tag{8}$$

Defining the exponential term of Eq. (8) as potential function [4,17]

$$\boldsymbol{\Phi}(\sigma, \mathrm{x}) = \zeta + \sum_i \sigma_i x_i + \sum_{ij} \sigma_{ij} x_i x_j + \sum_{ijk} \sigma_{ijk} x_i x_j x_k + \sum_{ijkl} \sigma_{ijkl} x_i x_j x_k x_l + \cdots \tag{9}$$

σ in left term represents vector, and σ in right term represents scalar. ζ and σ are both parameters produced by Lagrange optimization. Accordingly, by transformation of $x_i = \sum_k \psi_{ki}\xi_k$, Eq. (9) can be changed as [4,17]

$$\overline{\boldsymbol{\Phi}}(\lambda, \xi) = \zeta + \sum_k \lambda_k \xi_k^2 + \cdots \tag{10}$$

As general analyses, ξ is normalized by temperature difference in any reference point chosen. In applications, ξ is usually normalized by temperature difference between outside environment and liquid bulk.

Stable modes ($\lambda_k < 0$) represent that the bubbles cannot grow up and form, and unstable modes represent the unstable modes ($\lambda_k > 0$) stand for the formations of bubbles. Considering the identifiable contributions of stable modes and unstable modes, the potential function can be decomposed as [4,17]

$$\overline{\Phi}(\lambda,\xi) = \zeta + \overline{\Phi}_u(\lambda_u, \xi_u) + \overline{\Phi}_s(\lambda_u, \lambda_s; \xi_u, \xi_s) \tag{11}$$

where

$$\overline{\Phi}_u(\lambda_u, \xi_u) = \sum_u \lambda_u \xi_u^2 + \sum_{uu'u''} \lambda_{uu'u''} \xi_u \xi_{u'} \xi_{u''} + \sum_{uu'u''u'''} \lambda_{uu'u''u'''} \xi_u \xi_{u'} \xi_{u''} \xi_{u'''} + \cdots \tag{12}$$

$$\begin{aligned}\overline{\Phi}_s(\lambda_u, \lambda_s; \xi_u, \xi_s) &= \sum_s \big(-|\lambda_s|\xi_s^2\big) + \sum_{suu'} 3\lambda_{suu'} \xi_s \xi_u \xi_{u'} + \sum_{suu'u''} 4\lambda_{suu'u''} \xi_s \xi_u \xi_{u'} \xi_{u''} \\ &\quad + \sum_{ss'u} \lambda_{ss'u} \xi_s \xi_{s'} \xi_u + \sum_{ss's''} \lambda_{ss's''} \xi_s \xi_{s'} \xi_{s''} \\ &\quad + \sum_{ss'uu'} \lambda_{ss'uu'} \xi_s \xi_{s'} \xi_u \xi_{u'} + \sum_{ss's''u} \lambda_{ss's''u} \xi_s \xi_{s'} \xi_{s''} \xi_u \\ &\quad + \sum_{ss's''s'''} \lambda_{ss's''s'''} \xi_s \xi_{s'} \xi_{s''} \xi_{s'''} + \cdots\end{aligned} \tag{13}$$

Therefore

$$\rho(\mathbf{x}) = \rho(\xi_u, \xi_s) = \exp\big[\overline{\Phi}(\lambda, \xi)\big] = \rho(\xi_u)\rho(\xi_s/\xi_u) \tag{14}$$

According to statistical theory, Eq. (14) is satisfied at any conditions, for $\rho(\xi_u, \xi_s)$ is joint probability and $\rho(\xi_s/\xi_u)$ is conditional probability. Eq. (14) does not mean that event ξ_u and event ξ_s/ξ_u must be independent from each other [17]. Defining

$$\int \exp\big(\overline{\Phi}_s\big)\,\mathrm{d}^{N_s}\xi_s = g(\xi_u) = \exp\big[-h(\xi_u)\big] \tag{15}$$

N_s is the number of stable modes. Obviously

$$W_s = h(\xi_u) + \overline{\Phi}_s \tag{16}$$

$$W_u = \zeta + \overline{\Phi}_u - h(\xi_u) \tag{17}$$

Then

$$\rho(\xi_u) = \exp(W_u) \tag{18}$$

$$\rho(\xi_s/\xi_u) = \exp\big[W_s(\xi_s/\xi_u)\big] \tag{19}$$

Eqs. (18) and (19) can be regarded as the solutions of Fokker–Planck equations, which are equivalent to following Langivan equations

$$\dot{\xi}_u = \lambda_u \xi_u + S_u(\xi_u, \xi_s) + F_u(t) \tag{20}$$

$$\dot{\xi}_s = \lambda_s \xi_s + T_s(\xi_u) + F_s(t) \tag{21}$$

Clearly, for W_u and W_s are known, the functions S_u and T_s can be decided. Langivan equations are dynamic. The force $F(t)$ often appears in Langivan equation, and reflects stochastic effects, which put system to jump from one state to another [17]. In fact, we surprisingly obtained the typical evolution dynamic equations for boiling systems, by which self-organization of boiling systems can be investigated.

3. Dynamic analysis of bubble interactions in the framework of statistical mechanics

Above analyses facilitate an elegant way to investigate bubble interaction. We then consider multiple sites/bubbles interaction in a general way. As shown in Eqs. (20) and (21), interactions among active sites or bubbles are reflected through only one variable—normalized temperature difference in chosen region, which is directly related to the bubble dynamic parameters, such as radius variable r. Equations with form similar to Eqs. (20) and (21), which describe the dynamic interaction of all possible bubbles, are written as

$$\dot{\xi}_i = -\lambda_i \xi_i + f_i(\xi_1, \xi_2, \ldots, \xi_n) \quad (i = 1, 2, \ldots, n) \tag{22}$$

f_i is a function of multiple parameters when considering the interactions among numerous possible bubbles. Coefficients λ_i indicate damping effect. During the competitive processes controlled by this set of equations, controlling and controlled compromise prevails if coefficient λ_i differs each other. More specifically, the development of modes with small damping coefficients (mean small resistances) will dominate the development of modes with relatively large damping coefficients (mean large resistance). In fact, in classical quantum mechanics realm, according to well-known adiabatic elimination principle [17], for the interacting multi-elements system described by the kind of equations with form like Eq. (22), the self-organized evolving process prevails, providing that the damping coefficients λ_i are non-uniform. Let us do more detailed analyses. According to the magnitude of coefficients λ_i, Eq. (22) can be divided into some groups as was done in part 2.2: equations of the weakly damped modes with small λ_i denoted as $i = u = 1, 2, \ldots, m$, and equations of the stable modes with relatively large λ_i denoted as $i = s = m+1, m+2, \ldots, n$. For possible bubbles with stable modes (large λ_i) whose growths will be controlled by possible bubbles with weakly damped modes (small λ_i), in other word, these controlled bubbles cannot grow and $\dot{\xi} = 0$, and hence Eqs. (22) can be changed as

$$\lambda_s \xi_s = f_s\big(\xi_1, \xi_2, \ldots, \xi_m, \xi_{m+1}(\xi_i), \ldots, \xi_n\big) \tag{23}$$

This is a kind of self-organized process. In physical perspective, it can be imaged that when wall temperatures reach certain values, first group of sites/bubbles ($i = u = 1, 2, \ldots, m$) with small damping coefficients will be activated. Then, by interactions, the development of sites/bubbles ($i = u = 1, 2, \ldots, m$) will control and dominate the development of sites/bubbles ($i = s = m+1, m+2, \ldots, n$) with relatively large damping coefficients. If the damping coefficients satisfy

$$\lambda_1 \gg \lambda_2 \gg \lambda_3 \gg \cdots \tag{24}$$

the terms related to ξ_1 can be eliminated without affecting the other terms. The terms related to ξ_2 are then eliminated in succession until only one variable remains. When one mode does not dominate the other modes, flashing will occur. Flashing is likely to occur when

$$\lambda_1 \approx \lambda_2 \approx \lambda_3 \approx \cdots \approx \lambda_n \tag{25}$$

But the flashing only occurs on the special occasion such as strong or fast heating, or some other special cases that can make parameters of heating surface zone be uniform as soon as possible, which can confirm condition of Eq. (25) to satisfy. In general cases, for the existence of all kinds of stochastic factors in boiling system, the wall parameters are always non-uniform. In most nucleating processes in a boiling system, as described above, only one or a few sites in a specific unit will become unstable while most other sites will remain damped. The disturbance is induced by wall parameters, which affect the damped modes.

Sometimes we refer to show self-organized process through variable of bubble like radius r. In the following section we will show the above analysis is equivalent to that based on bubble radius. For an arbitrary boiling system, bubble growth rate can be rewritten by bubble radius as

$$\dot{r}_j = h_j(r_1, r_2, \ldots, r_n) \tag{26}$$

where h_j is a nonlinear function of $r_1, r_2, r_3, \ldots, r_n$. The function h takes the interactions among active sites or bubbles into account. For a steady-state solution of Eq. (26) given by r_j^0, the right side of Eq. (26) is dependent on a group of parameters $\beta_1, \beta_2, \beta_3, \ldots, \beta_n$, which are chose in such a way that r_j^0 stands for stable values. The origin of the r coordinate system can be shifted so that

$$r_j^0 = 0 \tag{27}$$

For small perturbations, the unsteady results can be described by

$$r_j(t) = r_j^0 + u_j(t) \tag{28}$$

$$\vec{r}\,(t) = \vec{r}^{\,0} + \vec{u}\,(t) \tag{29}$$

where r is decomposed into a steady-state value and a perturbation. If the system is stable and u_j is sufficiently small, Eq. (26) is linearized by substituting Eq. (28) into Eq. (26) so that

$$\dot{u}_j = \sum_{j'} L_{jj'} u_{j'} \tag{30}$$

Matrix L_{jj} is a function of $\vec{r}^{\,0}$ and $\beta_1, \beta_2, \beta_3, \ldots, \beta_n$. Eq. (30) is written as

$$\dot{\vec{u}} = L\,\vec{u} \tag{31}$$

Eqs. (30) or (31) is the first order ordinary differential equation with the solution

$$\vec{u} = \vec{u}^{\,(\mu)}(0)\exp(\lambda_\mu t) \tag{32}$$

here λ_μ are the eigen-values

$$\lambda_\mu\,\vec{u}^{\,(\mu)}(0) = L\,\vec{u}^{\,(\mu)}(0) \tag{33}$$

$\vec{u}^{\,(\mu)}(0)$ is the right eigen-vector, and the generalized solution of Eqs. (30) or (31) is the superposition of Eq. (32)

$$\vec{u} = \sum_{\mu} \xi_\mu \exp(\lambda_\mu t)\,\vec{u}_\mu(0) \tag{34}$$

ξ_μ can be considered as normalized temperature difference variable. Introducing the left eigen vector $\vec{v}^{\,(\mu)}$ and requiring that

$$\lambda_\mu\,\vec{v}^{\,(\mu)} = \vec{v}^{\,(\mu)} L \tag{35}$$

If the system is stable, the real parts of λ_μ are all negative. If non-linear effects are considered, Eq. (31) will have the form

$$\dot{\vec{u}} = L\,\vec{u} + \vec{N}(\vec{u}) \tag{36}$$

$\vec{N}(\vec{u})$ represents the nonlinear contribution. If u is still expressed in the form given by Eq. (34), then the appropriate expression $\xi(t)$ must be found so that the following relation is satisfied when left multiplying Eq. (36)

$$\left\langle \vec{v}^{\,(\mu)}, \vec{u}^{\,(\mu')} \right\rangle = \delta_{\mu\mu'} \tag{37}$$

Eq. (36) simply becomes

$$\dot{\xi}_\mu = \lambda_\mu \xi_\mu + g_\mu(\xi_1, \xi_2, \ldots, \xi_n) \tag{38}$$

where $g_\mu = \langle \vec{v}^{\,(\mu)}, \vec{N}(\sum_\mu \xi_\mu\,\vec{u}^{\,(\mu)}) \rangle$.

Eq. (38) has the same form as Eq. (22). The parameters $\beta_1, \beta_2, \beta_3, \ldots, \beta_n$, can be modified to destabilize Eq. (38), so that one or a few λ_μ are equal to or greater than zero with the others have a negative real part and thus are relating to the damping modes. If mode ξ_r for $\mathrm{Re}(\lambda_\mu) \geqslant 0$ is sufficiently greater than other modes, then adiabatic elimination and self-organization work.

Nucleate boiling is a self-organized dynamic process, which means that nucleate boiling system is a typical ordered dissipative structure [3]. The self-organized analysis provides a clear picture of the formation of this kind of ordered dissipative structure.

Then let us discuss how heat flux is distributed, which is more important for the understanding of self-organization. According to above dynamic analysis, and considering the decompositions of $J(\xi_u, \xi_s)$, Eq. (3) can be changed as

$$\overline{J} = \int_{\xi_u,\xi_s} \prod_s \rho_s(\xi_s/\xi_u)\rho(\xi_u)\left[J(\xi_u) + \sum_s J_s(\xi_s/\xi_u)\right] \mathrm{d}\xi \tag{39}$$

It is known that

$$\int_{\xi_s} \left[\rho_s(\xi_s/\xi_u)\right] \mathrm{d}\xi_s = 1 \tag{40}$$

Eq. (39) can be changed as

$$\overline{J} = \int_{\xi_u} \rho(\xi_u) J(\xi_u) + \int_{\xi_{u,s}} \rho(\xi_u) \int_{\xi_s} \rho_s(\xi_s/\xi_u) J_s(\xi_s/\xi_u)\,\mathrm{d}\xi \tag{41}$$

That is

$$\overline{J} = \int_{\xi_\mathrm{u}} \rho(\xi_u) J(\xi_u) + \int_{\xi_u} \rho(\xi_u) \int_{\xi_{s,s}} \rho_s(\xi_s/\xi_u) J_s(\xi_s/\xi_u)\,\mathrm{d}\xi \tag{42}$$

When we normalize the control parameter such that the instability occurring at $\alpha = 0$, then λ_u and λ_s depend on α in the following manners.

$$\lambda_u = \alpha^k \tag{43}$$

$$\lambda_s = \lambda_s(0) + 0(\alpha) \approx \lambda_s(0) \tag{44}$$

Where k is some positive number. Obviously, λ_u depends very sensitively on α, whereas λ_s depends only weakly on it because the leading term is a non-vanishing constant. Similarly, the functions S_u, T_s, F_u, and F_s depend also only weakly on α.

By introducing the new variables, we can eliminate the dependence of the probability distribution of the enslaved variables on ξ_s so that J_s becomes independent of ξ_u, therefore, in Eq. (42) we may perform the integration over ξ_u in the second term of the right hand. We thus obtain that

$$\overline{J} = J_u + \sum_s J_s \tag{45}$$

Where the second part does not depend on α, at least in the present approximation. Therefore, the flux change close to the instability point is governed by that of unstable sub-elements or modes alone

$$\overline{J}(\alpha_1) - \overline{J}(\alpha_2) \approx J_u(\alpha_1) - J_u(\alpha_2) \tag{46}$$

Eq. (46) provides clear physical picture: by self-organized process, only unstable sub-elements or modes with stronger ability can get flux and develop. For open complex boiling system that consists of many subsystems, some of them can use the heat flux better. In other words, heat flux

is concentrated in one or a few subsystems or modes, which may dominant the macroscopic behavior of the whole system.

4. Spatial correlation induced by bubble interactions

4.1. Interaction spatial correlation and fractal dimension

The above analyses show that heat flux is concentrated in one or a few sub-elements or modes, though a lot of possible modes exist simultaneously. In this way, we have the expression

$$\overline{J_n}(\alpha_1) - \overline{J_n}(\alpha_2) = J_{un}(\alpha_1) - J_{un}(\alpha_2) \sim h_n l^2 \xi_n \tag{47}$$

To consider bubble interaction spatial correlation, let us discuss heat flux interaction among different modes in all kind of spatial scales. We consider the heat flux from modes of last scale, heat flux increase in this scale and heat flux to next scale, the cascade heat flux balance then gives

$$\xi_S \frac{\mathrm{d}}{\mathrm{d}z} \int_0^l u \,\mathrm{d}y - a\left(\frac{\partial \xi}{\partial y}\right)\Big|_w = \frac{\mathrm{d}}{\mathrm{d}z} \int_0^l \xi u \,\mathrm{d}y \tag{48}$$

For arbitrary u and ξ distribution, Eq. (48) usually yields

$$h_n \sim l^{-p} \tag{49}$$

Depending on actual distribution of u and ξ, p, as a specific parameter ranging from 0 to 1.

Considering that driving forces are chosen as the temperature difference between heater and bulk of system, we can derive the fractal structure (Note: fractal structure is a kind of patch structure, and fractal geometry was developed by Mandelbrot to describe the structure with fractal dimension [18]) describing the distribution of active sites or bubbles in following way

$$\frac{\Omega_n \Delta J}{\Omega_{(n-1)} \Delta J} = \frac{\Omega_n}{\Omega_{(n-1)}} \sim \frac{J_{un}(\alpha_1) - J_{un}(\alpha_2)}{J_{u(n-1)}(\alpha_1) - J_{u(n-1)}(\alpha_2)} \sim \frac{h_n l_n^2 \xi_n}{h_{(n-1)} l_{(n-1)}^2 \xi_{n-1}} \sim \left[\frac{l_n}{l_{(n-1)}}\right]^{(2-p)} \tag{50}$$

Parameter $2 - p$ is fractal dimension, whose value indicates spatial distribution of active sites or bubbles. Fractal dimension physically embodies dynamics features of evolutional complex boiling systems. Once we have the u and ξ distribution (though determining u and ξ distribution is not an easy job), we can determine the distribution density of active sites or bubbles. Here, we roughly give two examples, if u and ξ have trivial polynomial distribution, which may correspond to case of laminar flow and heat transfer, we have $p = 1/2$ and $2 - p = 3/2$. If u and ξ have logarithmic distribution, which may correspond to the case of turbulent flow and heat transfer, we have $p = 1/5$ and $2 - p = 9/5$. This result is in agreement with experimental observations. During high heat flux, flow induced by bubble is very intense, more like violent turbulent flow, so we have a high density of active sites. While during low heat flux, flow induced by bubble is less intense than violent turbulent, we have a low density of active sites.

4.2. Engineering instructions

Enhancement of boiling heat transfer by means of artificial cavities is a very important aspect of heat transfer research because boiling occurs in a wide variety of industrial applications. The distribution of artificial cavities is a key factor for optimizing and controlling boiling heat transfer. The present investigations show that interactions among active sites or bubbles result in stochastic variations of local parameters and corresponding complex dynamic behaviors. Placing artificial cavities on a theoretically optimized distribution of active points can greatly improve the nucleate boiling heat transfer efficiency. The foregoing investigation results in a natural theoretical distribution of active sites under the optimization that maximizes heat flux. It is shown that the optimal distribution of active sites is dependent on the behavior of the active site itself. After knowing the feature of artificial site and heating condition, which mean that we can determine the distribution of u and ξ for the artificial site, we then can decide its distribution. The result may provide some instructions for design and distribution of artificial cavities on heating plates.

5. Conclusions

Bubble interaction is a classical puzzle during past decades. This paper proposed a novel theoretical analysis on the interactions among active sites or bubbles and the self-organizing effect among bubbles was revealed. The present studies may give more reasonable mechanistic descriptions of nucleation in boiling systems. The possible application is correspondingly discussed. The present investigation, although preliminary, provides a renewed theoretical effort to understand the underlying mechanisms of nucleate boiling.

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